SOLUTION TO QUIZ 2

MATH 241

Find all the solutions to the equation $\sin z = 2i \cos z$.

Proof. The equation is equivalent to $\frac{e^{iz}-e^{-iz}}{2i} = 2i \cdot \frac{e^{iz}+e^{-iz}}{2}$. So $e^{iz} - e^{-iz} = -2e^{iz} - 2e^{-iz}$, $3e^{iz} = -e^{-iz}$, $e^{2iz} = -\frac{1}{3}$.
So $2iz = \ln \frac{1}{3} + i(\pi + 2n\pi)$, eventually we get

$$z = -\frac{1}{2}i \ln \frac{1}{3} + \frac{1}{2}(\pi + 2n\pi) = \frac{1}{2}(\pi + 2n\pi) + (\frac{1}{2}\ln 3)i \quad n \in \mathbb{Z}$$

$\square$